Subject: Mathematics   **Course ID: 62117**

Course Code: SH/MTH/604/DSE-4   **Course Title: Mathematical Modelling**

Full Marks: 40   **Time: 2 Hours**

The figures in the margin indicate full marks

Symbols and Notations have their usual meaning

1. Answer any **five** from the following questions: $2 \times 5 = 10$

(a)   Determine the interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n}$.

(b) Find $L\{f(t)\}$, where L is the Laplace operator and $f(t) = t^n$.

(c) Show that $L(e^{-t}\cos t) = \frac{s+1}{s^2+2s+2}$.

(d) Write down the Bessel Equation of order μ.

(e) Define Queue in a Queuing System?

(f)   What is Sensitivity analysis?

(g) Write down the Canonical form of Simplex method.

(h) What is harbor System of Queuing   Model?

2. Answer any **four** from the following questions: $5 \times 4 = 20$

(a)   Find Power series solution of Legendre's equation.

(b)   Solve the initial value problem $y'' + 7y' + 12y = 0;\ y(0) = 1\ and\ y'(0) = 2$ by Laplace transform method.

(c)   The optimal table of the LPP

   $Max\ Z = 2\,x_1 + 3\,x_2 + 0\,x_3 + 0x_4$

   Subject to the constraints

   $x_1 + 2\,x_2 + x_3 = 2$

   $2\,x_1 - x_2 + x_4 = 3$

   $3\,x_1 + x_2 + x_5 = 5$

   $x_1, x_2, x_3, x_4, x_5 \geq 0;$

   where x_3 , x_4 , x_5 are slack variables, is given below.

Cost →				2	3	0	0	0
Variable →				x_1	x_2	x_3	x_4	x_5
Table No.	C_B	Basic variable	X_B	α_1	α_2	α_3	α_4	α_5
0	3	x_2	1/5	0	1	2/5	-1/5	0
	2	x_1	8/5	1	0	1/5	2/5	0
	0	x_5	0	0	0	-1	-1	1
		$c_j - Z_j$		0	0	-8/5	-1/5	0

Using the sensitivity analysis, find the solution to the new problem obtained by deleting the third constraint $3x_1 + x_2 \leq 5$.

(d)   Solve using simplex method

$Maximize\ Z = 3\,x_1 + 2\,x_2 + x_3$

Subject to the constraints

$2\,x_1 + x_2 + x_3 \leq 150$

$2\,x_1 + 2\,x_2 + 8x_3 \leq 200$

$2\,x_1 + 3\,x_2 + x_3 \leq 320$

$x_1, x_2, x_3 \geq 0.$

(e)   In an infinite storage queue system with one server (M/M/1): (FCFS/∞)

Derive the following expressions (i) Expected number of customer in the system,

(ii) Expected number of customer in the queue, (iii) Expected waiting time in the system. 2+2+1

(f)   Find $L^{-1} \left[-\frac{2}{s^2} + \frac{1}{s+1} \right]$.

3. Answer any **one** from the following questions: $10 \times 1 = 10$

(a) (i) Show that $L\{y''\}(s) = s^2 Y(s) - sy(0) - y'(0)$.

(ii) State and Prove the Convolution theorem of Laplace Transform. $5 + 5$

(b) (i) If $a \equiv b \bmod n\ \ and\ \ \ c \equiv d \bmod n\ \ then$ show   that

$a + c \equiv b + d \ \bmod \ n$ and $ac \equiv bd \ \bmod \ n.$

(ii) Find $17^{341} \bmod 5.$ $5 + 5$