B.SC. FOURTH SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42118**

Course Code: SP/MTH/401/C-1D **Course Title: Differential Equations and Vector Calculus**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Unless otherwise mentioned the symbols have their usual meaning

1. **Answer *any FIVE* of the following questions:** **2×5=10**

a) Illustrate by an example that a continuous function may not satisfy Lipschitz condition on a rectangle.

b) Find the regular and irregular singular points of the differential equation:

$$x^2(x-1)(x+2)^2\frac{d^2y}{dx^2}+x(x^2+2)\frac{dy}{dx}+(x+5)y=0$$

c) Calculate the values of $\vec{a}.\vec{b}$ and $\vec{c}\,.\,\vec{a}\times\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are defined by

$$\vec{a}=5\vec{\imath}-4\vec{\jmath}+\vec{k};\quad \vec{b}=-4\vec{\imath}+3\vec{\jmath}-2\vec{k};\quad \vec{c}=\vec{\imath}-2\vec{\jmath}-7\vec{k}$$

d) Convert the second order differential equation

$$2\frac{d^2y}{dx^2}-5\frac{dy}{dx}+4y=0$$

as a system of first order linear differential equations.

e) If $\vec{r}(t)=t\,\vec{\imath}-t^2\vec{\jmath}+(t-1)\,\vec{k}$ and $\vec{s}(t)=2t^2\vec{\imath}+6t\,\vec{k}$, show that $\int_0^2\vec{r}\,.\,\vec{s}\,dt=12$

f) Find the particular solution of the differential equation $y''-2y'=e^x\sin x$

g) Use Wronskian to show that the functions $f(x)=x,\ g(x)=x^2,\ h(x)=x^3$ are independent.

h) Find the values of n for which $\nabla^2r^n=0$, where r is the position vector of a point.

2. **Answer *any FOUR* of the following questions:** **5×4=20**

a) State the Lipschitz Condition. Show that $f(x,y)=xy^2$ satisfies Lipschitz condition on the rectangle $|x|\le 1,|y|\le 1$, but does not satisfy Lipschitz condition on the region

$$|x|\le 1,|y|<\infty.$$

b) Solve the differential equation arising in an electric circuit problem

$\left(LD^2 + RD + \frac{1}{C}\right)Q = E_0 \sin \omega t$, where $D = \frac{d}{dt}$; l, R, C, E_0 and w are given constants and $Q(0) = Q'(0) = 0$.

c) Solve the equation $(D^2 - 2D + 1)y = xe^x$ by the method of undetermined coefficients.

d) Find the general solution of the linear autonomous system

$$\frac{dx}{dt} = 3x + y, \qquad \frac{dy}{dt} = 3x - y.$$

Determine the nature of the critical point of the system and also comment on the stability.

e) Evaluate $\iiint (2x + y)dV$,where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$, and $z = 0$.

f) Verify Green's theorem in the plane for $\oint (2x - y^3)dx - xy\, dy$ on the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

3. Answer *any ONE* of the following questions: **10×1=10**

a) (i) Verify that $y_1 = x$ and $y_2 = x^2 - 1$ are linearly independent solutions of the homogeneous equation $y'' - \left(\frac{2x}{x^2+1}\right)y' + \left(\frac{2}{x^2+1}\right)y = 0$. Then using the method of variation of parameters, find the general solution of the non-homogeneous differential equation $y'' - \left(\frac{2x}{x^2+1}\right)y' + \left(\frac{2}{x^2+1}\right)y = 6(x^2 + 1)$. 3+4=7

(ii) Evaluate: $\frac{1}{D^2-3D+2}xe^x$, where $D \equiv \frac{d}{dx}$. 3

b) (i) Find the power series solution in powers of $(x - 1)$ of the initial value problem

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0, y = 2$ and $\frac{dy}{dx} = 4$at $x = 1$. 6

(ii) Find the directional derivative of the divergence of $f(x, y, z) = xy\hat{\imath} + xy^2\hat{\jmath} + z^2\hat{k}$ at the point (2,1,2) in the direction of outer normal to the sphere $x^2 + y^2 + z^2 = 9$. 4

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