B.SC. SECOND SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 22118**

Course Code: SP/MTH/201/C-1B **Course Title: Real Analysis**

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning

1. Answer any five questions : **2×5=10**

(a) Let $S=(0,1]$ and $T=\left\{\frac{1}{n}: n=1,2,3,\dots\right\}$. Show that $S-T$ is an open set.

(b) Determine the limit of the sequence $\left\{\frac{n^2+1}{n^2}\right\}$.

(c) If $\{|u_n|\}$ is a null sequence then show that $\{u_n\}$ is also a null sequence.

(d) Prove that $lim \frac{1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n+1}}{2n+1}$ = 0.

(e) Examine whether the following sets are compact or not: $(0,1)$, $[0,1]$.

(f) Test the convergence of the series $\frac{1}{1.2^2}+\frac{1}{2.3^2}+\frac{1}{3.4^2}+\cdots$.

(g) Give an example of a sequence which is bounded but not convergent.

(h) Show that the set $\mathbb{Z}\times\mathbb{Z}$ is countable.

2. ***Answer any four questions:*** **5×4=20**

(a) (i) Define limit point of a set. Let $S=\{x_1,\ x_2,\dots,x_n\}$ be a finite subset of $\mathbb{R}$, show that S has no limit point.

(ii) Is least upper bound axiom true for the set of all rational numbers? Justify it. 3+2

(b) (i) Use Cauchy's general principle of convergence to prove that the sequence $\left\{\frac{n}{n+1}\right\}$ is convergent.

(ii) Use Sandwich theorem to prove that $lim(\sqrt{n+1}-\sqrt{n}) = 0$. 3+2

(c) (i) State Leibnitz's test on alternating series. Use Leibnitz's test to show that the series $1-\frac{1}{2}+\frac{1.3}{2.4}-\frac{1.3.5}{2.4.6}+$ ... is convergent.

(ii) Prove that $m\frac{1}{n}\{(a+1)(a+2)\dots(a+n)\}^{\frac{1}{n}} = \frac{1}{e}$, if $a>0$. 3+2

(d) (i) State Bolzano-Weiestrass theorem and verify for the set $S=\{1+\frac{(-1)^n}{n}: n\in \mathbb{N}\}$.

(ii) Let A,B be two subsets of $\mathbb{R}$. Then show that $(A\cap B)^o=A^o\cap B^o$. 3+2

(e) (i) Let S be a non-empty bounded subset of $\mathbb{R}$ with $\sup S = M$ and $\inf S = m$. Prove that the set $A = \{|x - y| : x, y \in S\}$ is bounded above with $\sup A = M - m$.

(ii) Prove that the union of an infinite number of closed sets in $\mathbb{R}$ is not a closed set. 3+2

(f) (i) Give an example of a monotone increasing sequence.

(ii) Test the convergence of the series $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \cdots$. 1+4

3. Answer any one question: **10×1=10**

(a) (i) Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{3}$ and $u_{n+1} = \sqrt{3u_n}$ for $n \geq 1$, converges to 3.

(ii) Find $limsup\{x_n\}$ and $liminf\{ x_n\}$ where $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$.

(iii) Show that the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is conditionally convergent. 5+2+3

(b) (i) If a series $\sum u_n$ converges then show that $\lim u_n = 0$.

(ii) If a sequence $\{x_n\}$ converges to l then prove that every subsequence of $\{ x_n\}$ is also convergent to l.

(iii) State the Archimedean property of $\mathbb{R}$ and using Archimedean property show that for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $\frac{1}{k} < \epsilon$.

(iv) Show that a finite set has no limit point. 2+3+3+2

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