SP-I/Mathematics-101/C-1A/19

B.Sc. 1st Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 12118 **Course Code : SP/MTH/101/C-1A**

Course Title : Calculus, Geometry and Differential Equations

Time 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Unless otherwise mentioned, notations and symbol is have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Find $\int_0^{\pi/2} \sin^8 x \cos^{10} x \, dx$.

(b) Find $\lim_{x\to 0}(\cos x)^{1/x^2}$.

(c) If the co-ordinate axes are rotated through an angle 45° without changing the origin, find the transformed equation for $x^2 - y^2 = a^2$.

(d) Find the general solution of $\frac{dy}{dx} + Ay = B$ where A, B are function of x alone.

(e) Find an integrating factor of the differential equation $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(x + xy^2)dy = 0$.

(f) Find the envelope of the family of straight line $x\cos\alpha + y\sin\alpha = a$, α is the parameter.

(g) Find the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$.

(h) Evaluate $\lim_{x\to 0}\frac{\sin x - x}{x^3}$.

2. Answer *any four* questions: 5×4=20

(a) State Leibnitz's theorem on successive derivatives. If $y = \log(x + \sqrt{1 + x^2})$, then show that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + ny_n = 0$. 1+4=5

(b) Reduce the equation $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ to its canonical torm and determine the type of the conic represented by it.

(c) Define singular solution of an ordinary differential equation. If y_1 and y_2 be solutions of the equation $\frac{dy}{dx} + P(x)y = Q(x)$ and $y_2 = y_1 Z$, then show that $Z = 1 + a \cdot \bar{e}^{\int (Q/y_1)dx}$, where a is an arbitrary consant. 1+4=5

(d) (i) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then show that $n(I_{n+1} + I_{n-1}) = 1$.

(ii) Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord. 3+2=5

(e) (i) Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

(ii) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being a parameter.

(f) Find the asymptotes of $x^3 + 2x^2y + xy^2 - x + 1 = 0$.

3. Answer *any one* question: 10×1=10

(a) (i) If $\lim_{x\to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, find a and the value of the limit.

(ii) If $Z_n = \int_0^{\pi/2} x^n \sin x \, dx$ $(n \geq 1)$, show that $Z_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)Z_{n-2}$.

(iii) Solve $xdx + ydy + \frac{xdy - ydx}{x^2+y^2} = 0$, given that $y = 1$ when $x = 1$. 3+4+3=10

(b) (i) The number of bacteria in a yeast culture grows at a rate proportional to the number present. If the population of a colony yeast bacteria triple in 1 hour, find the number of bacteria that will be present at the end of 5 hours.

(ii) Prove that no two generators of the same system of a hyperboloid of one sheet intersect.

(iii) Show that the straight line $\frac{l}{r} = A\cos Q + B\sin Q$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(A-e)^2 + B^2 = 1$. 3+4+3=10