B.SC. FIRST SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 12111**

Course Code: SH/MTH/101/C-1
Course Title: Calculus, Geometry & Differential Equations

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. **Answer *any five* of the following questions:** $\mathbf{2 \times 5 = 10}$

a) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{sin^2 x}\right)$.

b) Find the point on the conic $\frac{l}{r} = 1 - \cos\theta$, which has the smallest radius vector.

c) Is the relation $x^2 + y^2 = 25$ an implicit solution of the differential equation $y\frac{dy}{dx} + x = 0$? Justify your answer.

d) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point $(1,2,3)$.

e) Solve: $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

f) Find the area bounded by the curve $\sqrt{y} + \sqrt{x} = \sqrt{a}$ and the co-ordinate axes.

g) If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(y)$, then prove that $e^{\int f(y)dy}$ is an integrating factor of the equation $Mdx + Ndy = 0$.

h) Find the envelope of the family of the straight line $y = mx + \sqrt{a^2m^2 + b^2}$, where m being the parameter.

2. **Answer *any four* of the following questions:** $\mathbf{5 \times 4 = 20}$

a) (i) In a certain spreading of the Corona virus, the rate of increasing of active case is proportional to number of the present size. If it be found that the number of active cases is double in 4 days. Then use mathematical model to find the number of active cases at the end of 12 days.

(ii) Find the n^{th} derivative of $x^3 \log x$. 3+2

b) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Then prove that the locus of the foot of the perpendicular from origin O to the plane ABC is given by $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4r^2$.

c) Find the asymptotes of the curve: $x^3 - 4xy^2 - 3x^2 + 12yx - 12y^2 + 8x + 2y + 4 = 0$.

d) (i) Solve: $(e^x \sin y + e^{-y})dx + (e^x \cos y - xe^{-y})dy = 0$.

(ii) Reduce the differential equation $\frac{dy}{dx} = 1 - x(y-x) - x^3(y-x)^3$ to a linear form and hence solve it. 2+3

e) (i) Find the canonical form and determine the nature of the conic $4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$.

(ii) Show that the points $(1,2)$ and $(8,-6)$ are conjugate with respect to the conic $x^2 + xy + y^2 = 1$. 3+2

f) If $J_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \cos nx\,dx$, then show that $J_{m,n} = \frac{m(m-1)}{m^2-n^2} J_{m-2,n}$ and hence prove that $\int_0^{\frac{\pi}{2}} \cos^n x \cos nx\,dx = \frac{\pi}{2^{n+1}}$. 3+2

3. Answer *any one* of the following questions: $\mathbf{10 \times 1 = 10}$

a) (i) Show that the solution of a differential equation $\frac{dy}{dx} + Py = Q$ can be written as $y = \frac{Q}{P} - e^{-\int Pdx}\left[c + \int e^{\int Pdx} d\left(\frac{Q}{P}\right)\right]$, where c is a constant.

(ii) If $y = \cos(m \sin^{-1} x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

(iii) If a circle cuts the parabola $y^2 = 4ax$ at the four points $(at_i^2, 2at_i)$, $i = 1,2,3,4$. Then show that $t_1 + t_2 + t_3 + t_4 = 0$. 4+3+3

b) (i) Prove that $(x+y+1)^{-4}$ is an integrating factor of the differential equation $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ and hence solve it.

(ii) Find the point of inflection of the curve $x = (y-1)(y-2)(y-3)$, if any.

(iii) Find the volume of the solid generated by revolving the cardioids $r = a(1 - \cos\theta)$ about the initial line. 4+3+3

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